Computation, Machines and Formal Languages - Exam (spring 2003) Moed A

Lecturer: Prof. Michael Ben-Or.

Course number: 67521

Time: 3 hours.

Write your answers on the Hebrew exam sheet. If you wish, you can write your answers in (readable) English.

If the answer is yes/no, **write down the answer that you chose**.

Answer all the questions 1-6 and **one** of the questions 7-9.

Unless stated otherwise, you have to add a short explanation (within the designated area for each question) for your answers. Answers that will not be accompanied by explanations will not be credited. The explanation should include the main steps in the proof, do not get into technical details, notations etc. If your explanation is given by a contradicting example, you have to state the example together with a short explanation of the contradiction. If you have an example that contradicts an unproven but a reasonable assumption (such as $P \neq NP$), state the example, the reasonable assumption, and a short explanation of the contradiction.

Definitions and Notations: (identical to those we used in class)

For a language $\mathcal{L}$, we denote by $\overline{\mathcal{L}}$ the language $\Sigma^* \setminus \mathcal{L}$ (i.e. the complement of $\mathcal{L}$).

Complexity Classes:
REG is the class of regular languages.
CFL is the class of context-free languages.
R is the class of languages that are decidable by Turing Machines (TM).
RE is the class of languages that are recognizable by TM's.
P is the class of languages that are decidable by deterministic TM's in polynomial time.
NP is the class of languages that are decidable by non-deterministic TM's in polynomial time.
EXP is the class of languages that are decidable by deterministic TM's in exponential time.
$PSPACE$ is the class of languages that are decidable by deterministic TM's in polynomial space.
For a class of languages $\mathcal{C}$, we denote by $co\mathcal{C}$ the class of languages that their complements are in $\mathcal{C}$. that is, $co\mathcal{C} = \{L : \overline{L} \in \mathcal{C}\}$.

Computational problems:
$SUBSET-SUM = \{(S,t)$: S is a (multi) set of numbers that contains a subset that sums to $t\}$
$HAM-PATH = \{(G,s,t)$: G is a directed graph, that contains a path from s to t, that visits every node in the graph exactly once$\}$
We say that a Boolean formula is $k-cnf$ if it is of the form:

$$\bigwedge_i (a_1^i \vee a_2^i \vee \ldots \vee a_k^i)$$

where a_j^i is a variable or its negation.
$k-SAT = \{\phi : \phi$ is a satisfiable $k-cnf\}$

Reductions:
For two languages $\mathcal{L}_1, \mathcal{L}_2$, $\mathcal{L}_1 \leq_p \mathcal{L}_2$ denotes that there is a polynomial-time mapping reduction from $\mathcal{L}_1$ to $\mathcal{L}_2$.

1. (15 points) You are given the following nondeterministic finite automaton (see the Hebrew version).

   (a) Does the string 1101000 belong to the language of the automaton? (yes/no)

   You do not have to prove or explain your answer.

   (b) Draw a **Deterministic** finite automaton with a minimal number of states for the same language.
   Give a short explanation of the correctness and the minimality.

2. (15 points) Let us define the following languages over $\Sigma = \{a, b, c, d\}$:

$$L_1 = \{a^n b^k c^n d^k : n, k \geq 0\}$$

$$L_2 = \{a^n b^k | n \leq k \leq 2n\}$$

   For each one of the following languages, state whether it is in CFL or not.

   You do not have to prove or explain your answers.

   (a) L_1
   (b) $\overline{L_1}$
   (c) L_2
   (d) $\overline{L_2}$

3. (15 points) For each one of the following languages, write the smallest class that contains it out of the following list: R, RE, $coRE$, or none of these classes.

   (a) $\{\langle M \rangle : M$ is a TM and $011 \in L(M)\}$
   (b) $\{\langle M \rangle : M$ is a TM and $L(M)$ contains infinite number of words $\}$

4. (15 points) Given a language L over an alphabet Σ that does not contain the symbol $\sharp$, we define the language:

   $EXACT - HALF(L) = \{w_1 \sharp w_2 \sharp \ldots \sharp w_{2k} : k \geq 0, w_i \in \Sigma^*$, and for **exactly** k indices $w_i \in L\}$

   What is the smallest class from: $REG, CFL, P, NP, PSPACE$, that contains the following set of labgauges:

(a) $\{EXACT - HALF(L) : L \in REG\}$

(b) $\{EXACT - HALF(L) : L \in P\}$

(c) $\{EXACT - HALF(L) : L \in NP\}$

5. (15 points) Does the assumption (that we currently can't prove) that $HAM-PATH \in P$ implies:

   (a) $NP = coNP$? (yes/no)

   (b) $SUBSET - SUM \leq_p 2 - SAT$? (yes/no)

6. (15 points) Under the assumption (that we currently can't prove) that $NP = EXP$, is it true that:

   (a) $NP = coNP$? (yes/no)

   (b) $NP = P$? (yes/no)

7. (10 points) Prove that the following language is not in RE nor in $coRE$.

   $\{\langle M \rangle : M$ is a TM and $L(M) \in DTIME(n^3)\}$

   Hint: Show reductions from the halting problem and its complement. The output of the reductions should be a TM that either accepts a trivial language or a hard language.

8. (10 points) Prove that for every enumerator E that enumerates the descriptions of all the Turing Machines in some order $M_1, M_2, \ldots$, there must exist an index i such that,

$$L(M_i) = L(M_{i+1})$$

   Hint: Use the recursion theorem.

9. (10 points) We say that two Boolean formulas, ϕ, ϕ', on the same variables are equivalent, if every assignment that satisfies ϕ also satisfies ϕ' and vice versa.
   We say that ϕ is minimal if there is no equivalent formula ϕ' to ϕ, that appears before it in the lexicographic order.

   Show an interactive proof system for the following language:

   $Non - Minimal - Formula = \{\phi : \phi$ is **not** minimal $\}$

Give a short explanation why this proof system has all the requirements.

Hint: Use the interactive proof system for $\sharp SAT$ that was given in class (you do not have to describe the proof system itself).